Lifetime of the Be^{10} 3.37-MeV Level. II. Analysis*

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The $E2$ lifetime of $(1.4\pm0.3)\times10^{-13}$ sec for the first excited state of Be^{10} at 3.37 MeV is discussed from the viewpoint of the independent-particle model. This transition is particularly suitable for analysis in terms of collective effects because the prediction of the pure independent-particle model (no mixing of major shells) is almost independent of the value of the intermediate coupling parameter a/K in the range 3–5 that is acceptable from other points of view. In the parameterization of the collective effects that consists of maintaining the independent-particle model wave functions but introducing an effective charge factor x such that the proton charge becomes $(1+x)e$ and the neutron charge xe, we find $x=0.5\pm0.2$. A satisfactory account in terms of the pure independent-particle model is excluded, and powerful collective effects are demanded even at $A=10$. So large a value of x may throw some doubt on the use of simple independent-particle model wave functions as a realistic starting point for the discussion of $1p$-shell nuclei. A major uncertainty in the analysis is the effective mean square nuclear radius and some discussion is given of this. The speed of the $E2$ transition from the first excited state of B^{10} is discussed in the same spirit. Consistency with the results on Be^{10} is found only for $a/K=4$–5, $x=0.4$–0.7, and a rather large nuclear radius.

I. INTRODUCTION

THE preceding paper[1] describes the determination of the lifetime for the $E2$ transition to ground from the 2^+ first excited state of Be^{10} at 3.37 MeV. We find a mean lifetime of $(1.4\pm0.3)\times10^{-13}$ sec; $\Gamma_\gamma=(4.7\pm1.0)\times10^{-3}$ eV. It is now of interest to compare this result with various expectations.

The first comparison is with the Weisskopf units since it is in terms of these that the local systematics are most frequently expressed. As mentioned in the preceding paper,[1] the $E2$ Weisskopf unit for this transition in Be^{10} is $\Gamma_{\gamma W}=4.6\times10^{-4}$ eV. Our measured speed therefore represents 10 such Weisskopf units and thus is quite representative of the local systematics.

This comparison with the Weisskopf unit is crude and although it suggests that the transition is somewhat enhanced we cannot express this quantitatively without recourse to explicit calculations based on the independent-particle model (IPM). Before we can make a meaningful comparison with theory we must evaluate, as realistically as possible, $\langle r^2\rangle_p$, the mean-square radius of the radiating particles (which we take to be those of the $1p$ shell) since the $E2$ speed depends on the square of this quantity.

The only reliable information on the nuclear size in this region of the periodic table (reliable in the sense of not itself leaning heavily on some explicit model) comes from the scattering of fast electrons. These data refer, of course, to $\langle r^2\rangle_t$, the mean-square radius for the total proton content of the various nuclei concerned; they have recently been summarized in convenient form.[2] Uncertainty attaches to the deduced values of $\langle r^2\rangle_t$ firstly on account of the inaccuracies in securing the best fit between the experimental data and the predictions of a particular model and secondly on account of the fact that the value of $\langle r^2\rangle_t$ so deduced depends somewhat on the type of nuclear model adopted (harmonic oscillator, exponential, etc.). We take the values of, and uncertainties in, $\langle r^2\rangle_t$ as those quoted in the summary; these derive from the wide range of models considered. We must now extract $\langle r^2\rangle_p$ from $\langle r^2\rangle_t$. We have used for this the relation:

$$\langle r^2\rangle_s=\tfrac{3}{5}\langle r^2\rangle_p,$$

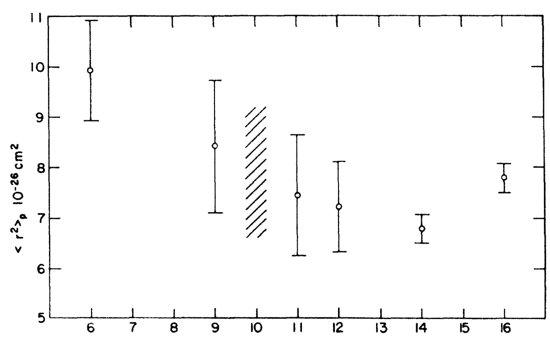

FIG. 1. Values of $\langle r^2\rangle_p$, the mean-square radius for the p-shell protons, derived from the total charge distribution as determined by electron scattering on the assumption: $\langle r^2\rangle_p=(5/3)\langle r^2\rangle_s$. The points refer to Li^6, Be^9, B^{11}, C^{12}, N^{14}, and O^{16}; the "error limits" reflect the range of values that derive from different models for the form of the charge distribution. The hatched band shows the value of $\langle r^2\rangle_p$ assumed for Be^{10} and used in the comparison between the experimental data and theory.

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[1] E. K. Warburton, D. E. Alburger, and D. H. Wilkinson, preceding paper, Phys. Rev. **129**, 2180 (1963).

[2] V. Meyer-Berkout, K. W. Ford, and A. E. S. Green, Ann. Phys. (N. Y.) **8**, 119 (1959).

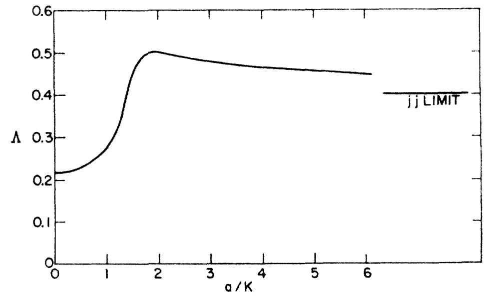

FIG. 2. The dimensionless transition strength Λ for the $E2$ transition in Be^{10} from the first excited state to ground as a function of the intermediate-coupling parameter a/K. This has been calculated using the force mixture given in the text. Points were computed at the integral values of a/K, 0–6; the line through them is by eye.

where the subscripts refer to the $1s$ and $1p$ shells, respectively, and the relation is appropriate to the harmonic oscillator model. This results in

$$\langle r^2\rangle_p=\frac{5Z}{5Z-4}\langle r^2\rangle_t.$$

Other models would give somewhat different relationships between $\langle r^2\rangle_t$ and $\langle r^2\rangle_p$ but this additional uncertainty is small compared with that already expressed in $\langle r^2\rangle_t$. Data are available for Li^6, Be^9, B^{11}, C^{12}, N^{14}, and O^{16}; we present the deduced values of $\langle r^2\rangle_p$, as a function of mass number, in Fig. 1. From this we conclude that a reasonable figure for Be^{10} is: $\langle r^2\rangle_p=(7.9\pm1.3)\times10^{-26}$ cm^2. (We do not discuss here the effect of the finite size of the proton's own charge distribution since the associated uncertainties are small compared with those that arise elsewhere in the analysis.)

II. INDEPENDENT PARTICLE MODEL

We now turn to the IPM. The transition has been calculated in intermediate coupling using the force mixture:

$$W=0.28,\quad M=0.45,\quad B=0.30,\quad H=-0.03.$$

L/K has been fixed at 6, a value that is found generally satisfactory in the $1p$ shell. It is a pleasing feature of these theoretical results, important for our present attempt to make a quantitative statement about enhancement in the $1p$ shell, that the theoretical pure IPM speed for this transition is a slowly varying function of a/K for the range of values of a/K (about 3 to 5) that can be regarded as acceptable for nuclei near $A=10$; this means that the theoretical speed cannot be adjusted through a wide range by varying a/K. This important point is illustrated in Fig. 2 where the theoretical speed is shown as a function of a/K in units of the dimensionless transition strength Λ of Lane and Radicati[3] which is effectively the transition speed in appropriately defined single-particle units.[4] It is also gratifying that Λ is large in the region of interest. This means that an experimental radiative width in excess of the theoretical is significant because it is not due to an accidentally small value of the theoretical speed associated with such things as the particular choice of force mixture and so on. These two points make this a particularly useful transition for seeking collective effects in light nuclei.

We must also ask how the theoretical speed changes if we depart from the pure IPM in the now conventional way and represent an admixture of explicitly collective motion (i.e., additional to that already implied by the antisymmetrization within the shell model) by giving to the protons the fictional charge $(1+x)e$ and to the neutrons the corresponding charge xe. The pure IPM has the value $x=0$ for this effective charge parameter and a significantly nonzero value for x is a demand for the recognition of collective motion in the sense in which we have defined it. The accounting for the collective motion in terms of the mixing of major shells or other language is an issue into which we shall not enter here. We may notice, however, that if this effective charge procedure is taken as one of weak coupling of the nucleons of the unfilled shell, that are used to construct the IPM wave function, to the underlying filled shells the approximation is not valid if the value of x demanded by it does not satisfy $x\ll Z_f/A_u$, where Z_f is the number of protons in the filled shell and A_u is the number of nucleons in the unfilled shell; in the present case this means $x\ll0.33$. If the procedure demands larger values of x than this we may have to regard the result as a criticism of the IPM wave functions themselves since they then need modification in a more

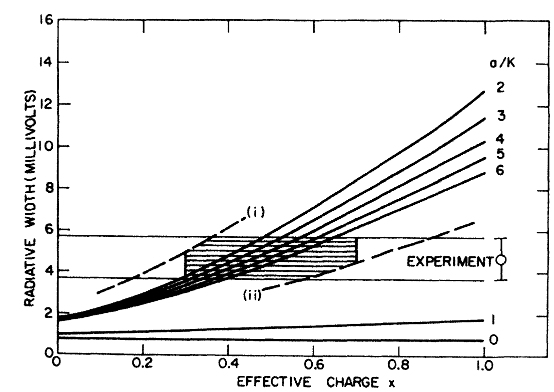

Fig. 3. The theoretical radiative width for the $E2$ transition in Be^{10} for various values of a/K as a function of the effective charge factor x [proton charge $(1+x)e$; neutron charge xe]. The solid lines are for $\langle r^2\rangle_p=7.9$ F^2. The dashed lines are: (i) $a/K=3$ and $\langle r^2\rangle_p=9.2$ F^2; (ii) $a/K=5$ and $\langle r^2\rangle_p=6.6$ F^2. The experimental result is shown and it is likely that the heavily shaded area, $x=0.5\pm0.2$, contains the correct representation of the $E2$ transition.

[3] A. M. Lane and L. A. Radicati, Proc. Phys. Soc. (London) **A67**, 167 (1954).

[4] The present calculation does not agree with that of A. N. Boyarkina and A. F. Tulinov [J. Exptl. Theoret. Phys. (U.S.S.R.) **36**, 353 (1959)] [translation: Soviet Phys.—JETP **9**, 244 (1959)]; in particular, their report that the matrix element vanishes in j-j coupling would seem to be in error.

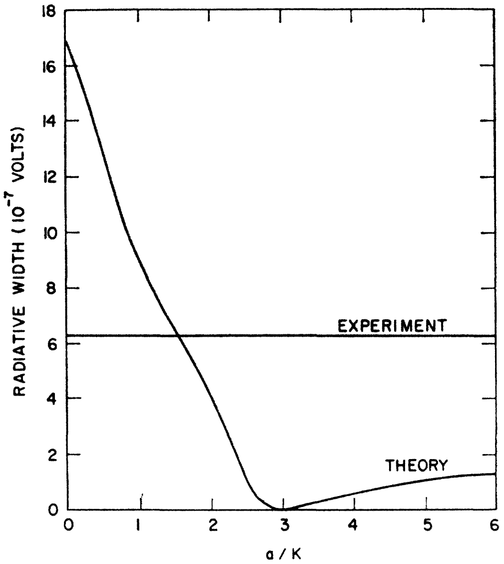

FIG. 4. Speed of the $E2$ transition in B^{10} from the first excited state to ground as a function of the intermediate coupling parameter a/K and for $\langle r^2\rangle_p=7.9$ F^2. This has been calculated using the force mixture given in the text. Points were computed at the integral values of a/K, 0–6, and also at $a/K=0.5$, 1.5, 2.5, and 3.5; the line through them is by eye.

thoroughgoing way than can be represented simply by the addition of the effective charge in a weak-coupling approximation. In this case the value of a/K that, in the pure IPM, best fits the experimental level scheme will itself be only a sort of parameterization of a more complicated situation, and we should perhaps not expect the same degree of internal consistency in its behavior within a given shell as we should if it had its literal significance nor be able to give a simple interpretation to that behavior.

We have $\Gamma_{\gamma\,\mathrm{theo}}=(1+\lambda x)^2\Gamma_i$, where Γ_i is the prediction of the pure IPM and where λ, as well as Γ_i, is a function of a/K and has also been calculated. The final situation is summarized in Fig. 3 where we show Γ_{theo} as a function of the effective charge for various values of a/K. The full lines correspond to $\langle r^2\rangle_p=7.9$ F^2. The two dashed lines have been obtained: (i) by taking the "lower limit" of $a/K=3$ together with the "upper limit" $\langle r^2\rangle_p=9.2$ F^2 to give the fastest reasonable theoretical expectation as a function of x, (ii) by similarly combining $a/K=5$ with $\langle r^2\rangle_p=6.6$ F^2 to give the slowest reasonable theoretical expectation. Also shown on Fig. 3 is the error band of the experimental value $\Gamma_{\gamma\,\mathrm{exp}}$. The heavily shaded region shows the area within which we feel the truth most probably lies. This corresponds to $x=0.5\pm0.2$.

This value of x shows that there exists very strong collective behavior even in so light a nucleus as $A=10$. It is quite inconsistent with $x\ll Z_f/A_u$ and so may throw doubt on the basis of the IPM representation, as we have just remarked.

It is now interesting to see whether the same set of parameters that we have arrived at here, namely: $a/K=3$–5; $\langle r^2\rangle_p=6.6$–9.2 F^2; $x=0.3$–0.7, is consistent with other $E2$ effects in the $A=10$ system. The only case that can be analyzed at the moment is the pure $E2$ transition between the 1^+ 0.72-MeV first excited state of B^{10} and the 3^+ ground state. The mean lifetime of this state is known very accurately to be (1.04 ± 0.02) nsec,[5] which corresponds to 3.1 Weisskopf units for a radius constant of 1.2 F; the transition is therefore strong and typical of the local systematics. The situation in relation to the IPM is shown in Fig. 4 which gives the theoretical speed Γ_i for the above force mixture and $\langle r^2\rangle_p=7.9$ F^2 as a function of a/K. We see that, unlike the favorable case in Be^{10}, the theoretical speed is here a strong function of a/K and so the situation is not clear cut. It has, however, an interesting aspect that is shown in Fig. 5. Here we display, as a function of a/K, the effective charge that is necessary to gain agreement between theory and experiment, as before, for the "upper and lower limits" $\langle r^2\rangle_p=6.6$ F^2 and 9.2 F^2. Theory and experiment agree within the stippled areas. The small region consistent with $a/K=3$–5 and $x=0.3$–0.7 as derived from the Be^{10} result is shown hatched. It seems that a rather large value of a/K and also a large nuclear size and large effective charge are needed to make the transitions consistent. We cannot immediately accept this conclusion. One point to notice is that the theoretical transition strength in the region of a/K of interest is low. Λ runs from 0 to 0.069 as a/K runs from 3 to 5 as compared with the large values for Λ found in the case of Be^{10} and displayed in Fig. 2. This means that the

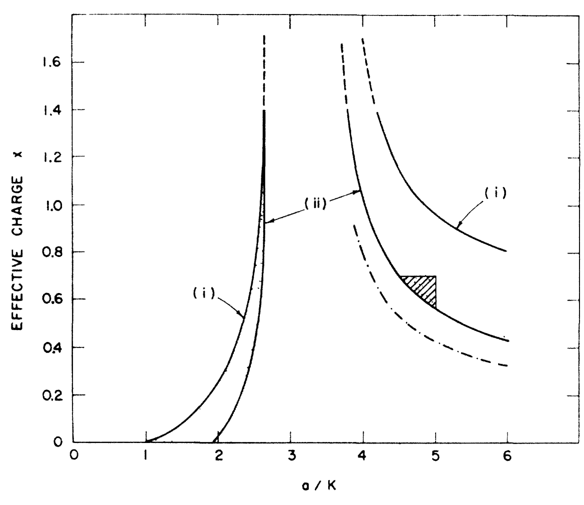

FIG. 5. The effective charge factor x required to give agreement between theory and experiment for the $E2$ transition in B^{10} as a function of a/K for the force mixture given in the text. The lines (i) are for the "small" nucleus $\langle r^2\rangle_p=6.6$ F^2 and the lines (ii) for the "large" nucleus $\langle r^2\rangle_p=9.2$ F^2. Theory and experiment agree within the stippled area. The hatched region is that portion of the stippled area consistent with $a/K=3-5$ and $x=0.5\pm0.2$ as derived from the transition in Be^{10}. The dash-dot curve is a portion of the results for the Kurath mixture and the "large" nucleus.

[5] J. Lowe, C. L. McClelland, and J. V. Kane, Phys. Rev. **126**, 1811 (1962).

result may well be rather sensitive to the force mixture used and other details. Kurath[6] has published results for the force mixture of 0.8 space exchange and 0.2 spin exchange. This results in a slightly stronger transition (by a factor of 1.3 to 1.4) in the region of interest. This is shown by the dash-dot line in Fig. 5 which is drawn for $\langle r^2 \rangle_p = 9.2$ F^2. This brings no change in the qualitative conclusion although a value of a/K of about 4 is now allowed. We note that Kurath's results cannot give agreement for $a/K < 3$ either. But it must be admitted that a substantially different force mixture may give an increased transition strength and so permit lower values for the various parameters than those implied by the restricted hatched area of Fig. 5. We must also ask if we may not use the lower branch of Fig. 5 and so gain agreement in the region $a/K \approx 2.5$. So low a value of a/K seems to be completely excluded by the level scheme if we are to continue to use the IPM at all.[7] Already at $a/K = 3$ the first excited, 1^+, state has come down to meet the ground state, the third excited, 1^+, state has crossed well below the second excited, 0^+ $T=1$, state and the fourth excited, 2^+, state has come down to meet the 0^+ $T=1$ state. (This is true for the force mixture used in the present lifetime calculations as well as that used by Kurath.)

We, therefore, conclude that to accommodate the level schemes and $E2$ transitions of Be^{10} and B^{10}, at least in the versions of the IPM discussed here, we must take a/K in the region 4 to 5 together with a large nuclear radius represented by $\langle r^2 \rangle_p$ greater than about 8 F^2 and a large effective charge x of about 0.4 to 0.7.

We finally note that this conclusion may run counter to the failure of the expected $5/2^-$ state of C^{13} and of N^{13} to show up below an excitation of 7 MeV.[8] This seems to require a value of a/K of not greater than 3.5 which contradicts the usually accepted result that a/K increases monotonically through the $1p$ shell. This may be due to the inadequacy of the "IPM plus effective charge" approach that is suggested by the large effective charge values discussed here or it may be due to the strong cancellations in the $E2$ transition in B^{10} shown in the vanishing of the matrix element in the critical region $a/K \approx 3$. But for this latter feature a lower value of a/K, in better agreement with the $A=13$ results, would be allowed. It may then also be possible to return to smaller values of $\langle r^2 \rangle_p$ and the effective charge. An investigation of the $E2$ situation in B^{10} for other force mixtures is most desirable.

III. CONCLUSIONS

(i) The speed of the 3.37-MeV Be^{10} transition as measured here shows that strong collective effects are present at $A=10$ and that the effective charge in the weak-coupling model has the value $x = 0.5 \pm 0.2$. (ii) This large value of the effective charge may throw some doubt on the significance of the simple IPM representation as a realistic starting point for the discussion of $1p$-shell nuclei. (iii) The speed of the 0.72-MeV B^{10} transition may demand an a/K value of greater than 4 and a value of $\langle r^2 \rangle_p$ greater than 8 F^2 together with a value of the effective charge $x = 0.4$–0.7, but these conclusions are rather unsure because the theoretical speed is low and so the result may be substantially different for other versions of the calculation.

[6] D. Kurath, Phys. Rev. **106**, 975 (1957). This paper reports that the matrix element changes sign twice in the region $a/K > 1.5$; we find only one such change. J. B. French and A. Fujii [*ibid.* **105**, 652 (1957)] have calculated two points in intermediate coupling which imply large oscillations of the matrix element (with at least three changes of sign) as a function of a/K. We find nothing corresponding to these in our work.

[7] D. Kurath, Phys. Rev. **101**, 216 (1956).

[8] A. Gallmann, D. E. Alburger, D. H. Wilkinson, and F. Hibou, Phys. Rev. **129**, 1765 (1963).